A PREDICTIVE MODEL FOR THE DESIGN OF FUNCTIONAL TEXTILES

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Summary: This report proposes a method for the design of a fabric for specified mechanical properties at multiple biaxial-stress states.

1 INTRODUCTION

Functional textiles have a wide variety of uses including large scale roof structures [1], medical applications [2], and as reinforcement for composite materials. Functional textiles are typically manufactured based on simplified engineering requirements (e.g. weight and uniaxial strength), with other properties (such as detailed analysis of stiffness) determined retrospectively through physical testing. The work presented here demonstrates a methodology for the design of bespoke functional textiles to meet detailed engineering requirements, with the focus on the biaxial response of flexible coated woven fabrics. The method employed uses a semi-analytical optimisation routine to determine the optimum fabric geometry and constituent material properties for detailed material stiffness requirements.

Previously developed mechanical 'unit cell' models have been shown to provide a good prediction of the response of architectural plain-weave fabrics under biaxial load, and have therefore formed the basis of the work [3, 4]. The derivatives of the unit cell equilibrium equations have been determined and this allows the fabric parameters to be optimised for a detailed set of biaxial and shear stiffness requirements at different stress levels. Initial validation using the model to design feasible, known fabrics has shown good results and demonstrated the potential utility of this approach.

2 SCOPE AND METHODOLOGY

2.1 Biaxial response

Coated architectural fabrics are employed in biaxial stress states and have "*negligible bending or compression stiffness*" [5] meaning loads are resisted through tension, and as such the model was required to work with biaxial input and output parameters. Therefore the response characteristics under biaxial load are considered to be the Young's moduli in both warp and weft directions (E_{11} and E_{22}) and the Poisons ratios of the fabric (ν_{12} and ν_{21}).

Whilst shear response under biaxial load "*is crucial in order to build double-curvature tensioned structures*"[6] the shear modulus (G) is not considered in the current version of this model as the response has been found to be dominated by the coating stiffness, currently modelled as linear. It is proposed that later versions of this model will include a module for the consideration of shear effects.

2.1 Sawtooth modelling

The sawtooth model developed by Menges and Meffert [7] and further developed and used by Bridgens [3, 4] was the basis of the work. It was chosen as it allowed for the possibility of truly predictive design, as the equations contain no factors that need to be derived through testing, and the equations themselves lend themselves to differentiation.

The method considers a unit cell of fabric as shown in Figure 3, and idealises this as a set of two orthotropic yarns that are perpendicular, as shown in Figure 1 and Figure 2.

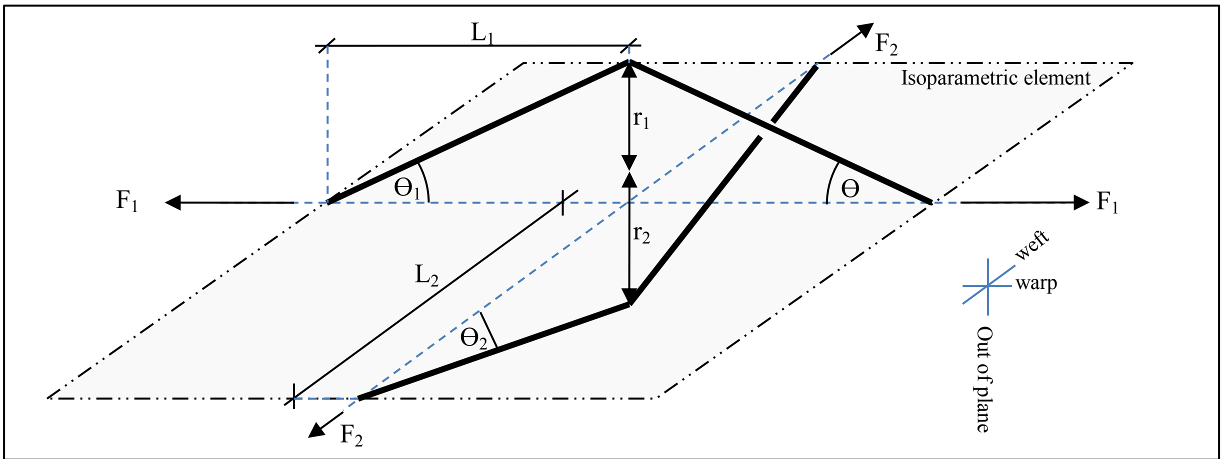

Figure 1: Fundamentals of the full sawtooth model with an Isoparametric Element representing the coating

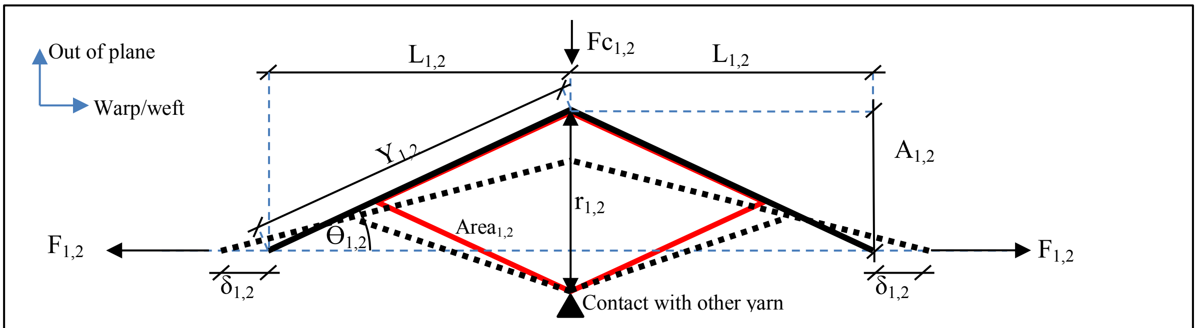

Figure 2: Further definitions within the unit cell

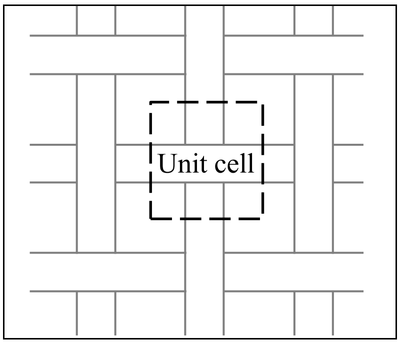

Figure 3: Unit Cell representation (Plain Weave)[3, 4]

Unlike the previously developed models the coating is represented by a single Isoparametric Plane Stress element as described by Cook, Malkus and Plesha [8]. This change was made in preparation for the analysis of shear response and the possibility of non-perpendicular geometry. The equations defining the response of the unit cell are therefore published as:

$$F_{k1,2} = 2 \cdot L_{2,1}\left(\frac{E_k}{1-{v_k}^2}\right)\left(\varepsilon_{1,2} + v_k\varepsilon_{2,1}\right)\left(1+\varepsilon_{2,1}\right) \quad , \tag{1}$$

$$Y'_{1,2} = Y_{1,2}\left[1 + \frac{F_{y1,2}}{2E_{y1,2}L'_{2,1}}\right] \quad ,$$

$$Area_{1,2} = 2w_{1,2}r_{1,2} \quad ,$$

$$w'_{1,2} = \frac{w_{1,2}}{L_{2,1}}L'_{2,1} \quad ,$$

$$r'_{1,2} = \frac{Area_{1,2}}{2 * {w_{1,2}}'} \quad ,$$

[3, 4]

constrained by the following equations which ensure geometric continuity and force equilibrium:

$$(r_1 + r_2) = (A_1 + A_2) \quad , \tag{2}$$

$$F_{c1} = F_{c2} \quad ,$$

$$F_{1,2} = \mathrm{F}_{\mathrm{y}1,2}\cos\theta'_{1,2} + \mathrm{F}_{\mathrm{k}1,2} \quad ,$$

[3, 4]

where the subscripts 1 and 2 refer to the warp and weft directions respectively. The subscripts k and y refer to the coating and yarn respectively. The apostrophe refers to a value after deformation. Other terms included are the yarn radius (r), the yarn length (L) (1/4 the yarn wavelength), force (F), yarn amplitudes (A), yarn widths (w) (1/2 the yarn width), the yarn cross-sectional area (Area), Young's Moduli (E), and yarn length (Y) (includes out-of plane distance).

3 RESULTS AND DISCUSSION

3.1 Construction of defining equations

Once the equations defining the unit cell are available it is possible to calculate the response characteristics of the fabric numerically, employing a finite difference method.

However, numerical perturbation does not lend itself to optimisation, which is necessary to design a bespoke fabric. To produce equations that can be used in conjunction with optimisation routines it is necessary to find the derivatives $\frac{\mathrm{dF}_{1,2}}{\mathrm{d}\varepsilon_{1,2}}$ (for $E_{11,22}$) and $\frac{\mathrm{dF}_{1,2}}{\mathrm{d}\varepsilon_{2,1}}$ (for $E_{12,21}$). The derivative $\frac{dF_{1,2}}{d\varepsilon_{1,2}}$ refers to the Young's modulus of the unit cell, and must be converted to the value for the whole fabric as shown in equation 3. The derivative $\frac{dF_{1,2}}{d\varepsilon_{2,1}}$ is needed to produce the Poisson's ratios, as shown in equation 4.

$$E_{11,22}{}^{unit\ cell} = E_{11,22}{}^{global} \times L_{2,1} \times 2 \tag{3}$$

$$-\frac{v_{12}}{E_{11}} = \frac{1}{E_{12}} \tag{4}$$

[1]

To find the derivatives the applied force was determined in terms of the unit cell variables, and strain as shown in equation 5. Equations 6 through 9 are then necessary to calculate further derivatives.

$$F_{1,2} = \frac{\left(F_{2,1} - F_{k2,1}\right)\left((r'_1 + r'_2) - L_{1,2}\left(1 + \varepsilon_{1,2}\right) \tan\theta'_{1,2}\right)}{L_{2,1}\left(1 + \varepsilon_{2,1}\right) \tan\theta'_{1,2}} + F_{k1,2} \tag{5}$$

$$F_{k1,2} = 2 \cdot L_{2,1}\left(\frac{E_k}{1 - {v_k}^2}\right)\left(\varepsilon_{1,2} + v_k\varepsilon_{2,1}\right)\left(1 + \varepsilon_{2,1}\right) \tag{6}$$

$$r'_{1,2} = \frac{r_{1,2}}{\left(1 + \varepsilon_{2,1}\right)} \tag{7}$$

$$\theta'_{2,1} = cos^{-1}\left(\left(1 + \varepsilon_{2,1}\right) \cos\theta_{2,1} - \left(\frac{\left(F_{2,1} - F_{k2,1}\right)}{2E_{2,1}L_{1,2}\left(1 + \varepsilon_{1,2}\right)}\right)\right) \tag{8}$$

$$\theta'_{1,2} = tan^{-1}\left(\frac{(r'_1 + r'_2) - L_{2,1}\left(1 + \varepsilon_{2,1}\right) \tan\theta'_{2,1}}{L_{1,2}\left(1 + \varepsilon_{1,2}\right)}\right) \tag{9}$$

To calculate the full derivatives it is necessary to find the partial derivatives for all the variables. There are numerous variables that are inter-related with relation to the defining equations expressed earlier (equations 1 and 2). As such equations 10 and 11 represent the calculation that needs to be performed to produce the required derivatives.

$$\frac{dF_{1,2}}{d\varepsilon_{1,2}}=\frac{\partial F_{1,2}}{\partial\varepsilon_{1,2}}+\frac{\partial F_{1,2}}{\partial\theta'_{1,2}}\cdot\frac{\partial\theta'_{1,2}}{\partial\varepsilon_{1,2}}+\frac{\partial F_{1,2}}{\partial r'_{1,2}}\cdot\frac{\partial r'_{1,2}}{\partial\varepsilon_{1,2}}+\frac{\partial F_{1,2}}{\partial r'_{2,1}}\cdot\frac{\partial r'_{2,1}}{\partial\varepsilon_{1,2}}+\frac{\partial F_{1,2}}{\partial\varepsilon_{2,1}}\cdot\frac{\partial\varepsilon_{2,1}}{\partial\varepsilon_{1,2}}+\frac{\partial F_{1,2}}{\partial F_{k1,2}}\cdot\frac{\partial F_{k1,2}}{\partial\varepsilon_{1,2}}+\frac{\partial F_{1,2}}{\partial F_{k2,1}}\cdot\frac{\partial F_{k2,1}}{\partial\varepsilon_{1,2}} \tag{10}$$

$$\frac{dF_{1,2}}{d\varepsilon_{2,1}}=\frac{\partial F_{1,2}}{\partial\varepsilon_{2,1}}+\frac{\partial F_{1,2}}{\partial\theta'_{1,2}}\cdot\frac{\partial\theta'_{1,2}}{\partial\varepsilon_{2,1}}+\frac{\partial F_{1,2}}{\partial r'_{1,2}}\cdot\frac{\partial r'_{1,2}}{\partial\varepsilon_{2,1}}+\frac{\partial F_{1,2}}{\partial r'_{2,1}}\cdot\frac{\partial r'_{2,1}}{\partial\varepsilon_{2,1}}+\frac{\partial F_{1,2}}{\partial\varepsilon_{1,2}}\cdot\frac{\partial\varepsilon_{1,2}}{\partial\varepsilon_{2,1}}+\frac{\partial F_{1,2}}{\partial F_{k1,2}}\cdot\frac{\partial F_{k1,2}}{\partial\varepsilon_{2,1}}+\frac{\partial F_{1,2}}{\partial F_{k2,1}}\cdot\frac{\partial F_{k2,1}}{\partial\varepsilon_{2,1}} \tag{11}$$

Unfortunately it can be shown that due to the interdependence of the variables it is not possible to produce a fully analytical answer to equations 10 and 11. To produce useable equations one value must be calculated iteratively, as shown in equation 12. This must be calculated independently using the equilibrium model each time a new value is required.

$$\frac{\delta\varepsilon_{1,2}}{\delta\varepsilon_{2,1}}=\frac{\Delta\varepsilon_{1,2}}{\Delta\varepsilon_{2,1}} \tag{12}$$

Whilst this is now a semi-analytical method the equations derived do still allow for optimisation to be used to design a bespoke fabric.

3.2 The method of optimisation

MATLAB [9] was used to produce an optimisation script for the minimisation of the defining equations. Internal functions were used to optimise the equations for a set of targets produced. The optimisation methodology is briefly summarised in Figure 4. The method initially uses a pattern search algorithm to refine the search 'area', and then uses an internal MATLAB search routine to find the "*minimum of* [a] *constrained nonlinear multivariable function*" [10]. If no perfect solution can be found then the script implements a gradually varying allowance of variation from the targets to allow a solution to be found. This could be changed to allow for accurate optimisation for some important targets, and 'as close as possible' optimisation for other targets of less significance to the designer.

Using a function that allows for multiple constraints is used to incorporate the constraint equations (equations 2). If no perfect solution is found then bounds are placed on the targets, and these are allowed to vary by a percentage. This allows the script to find results where no realistic solution would be possible.

Five sets of targets are used in the current model to demonstrate how the method can be used to design for multiple material properties for a single fabric at different loads. More targets could be implemented, however the current number demonstrates the method's utility without making any solution too difficult, or computationally expensive to find. The 'Shear Module' shown is currently in development.

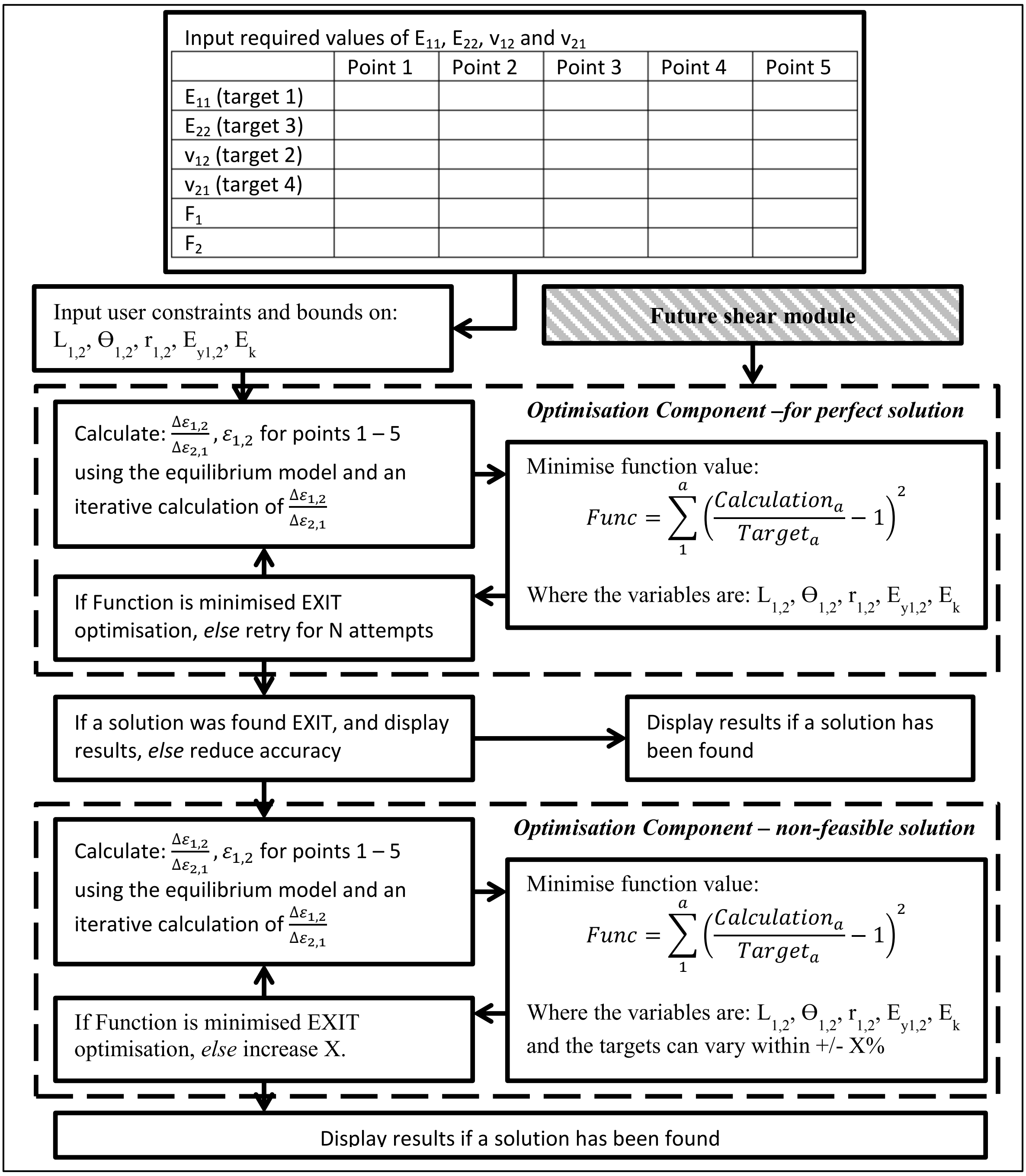

Figure 4: Flow chart to describe the optimisation process

3.3 Results for known feasible targets

To demonstrate the functionality of both the method of optimisation and the validity of the equations used an optimisation for a set of targets that were known to be feasible was performed.

The feasible targets were produced with the equilibrium model using a central finite difference method from the geometry shown in Table 1. The results of this finite difference method are shown in Table 2.

Table 1: Geometry used to find feasible targets and resultant optimised geometry

Variable	Geometry from which targets are calculated	Optimised geometry
A_1 (mm)	0.069	0.071
A_2 (mm)	0.207	0.190
Θ_1 (Rad)	0.106	0.116
Θ_2 (Rad)	0.189	0.183
L_1 (mm)	0.645	0.605
L_2 (mm)	1.082	1.022
r_1 (mm)	0.162	0.152
r_2 (mm)	0.114	0.107
w_1 (mm)	0.786	0.824
w_2 (mm)	0.673	0.920
E_1 (kN/m)	860	859
E_2 (kN/m)	710	703
E_k (kN/m)	30	33
v_k	0.3	0.3

Table 2: Feasible targets found at the applied loads P1 and P2.

	Point 1	Point 2	Point 3	Point 4	Point 5
E_{11} (target 1) (kN/m)	514	662	602	377	777
E_{22} (target 3) (kN/m)	444	554	510	551	484
v_{12} (target 2)	0.434	0.288	0.344	0.317	0.261
v_{21} (target 4)	0.374	0.241	0.291	0.431	0.180
P_1 (kN/m)	10	20	15	10	20
P_2 (kN/m)	10	20	15	20	10

The results of this are as expected, a near perfect solution is found quickly suggesting the equations appear to correlate well to the sawtooth method which is known to correlate well with the response of real fabrics. It should be noted that the start point of the optimisation was not the geometry used to find the targets; this ensured that the method was in fact finding a solution, and not succeeding having been given the correct geometry.

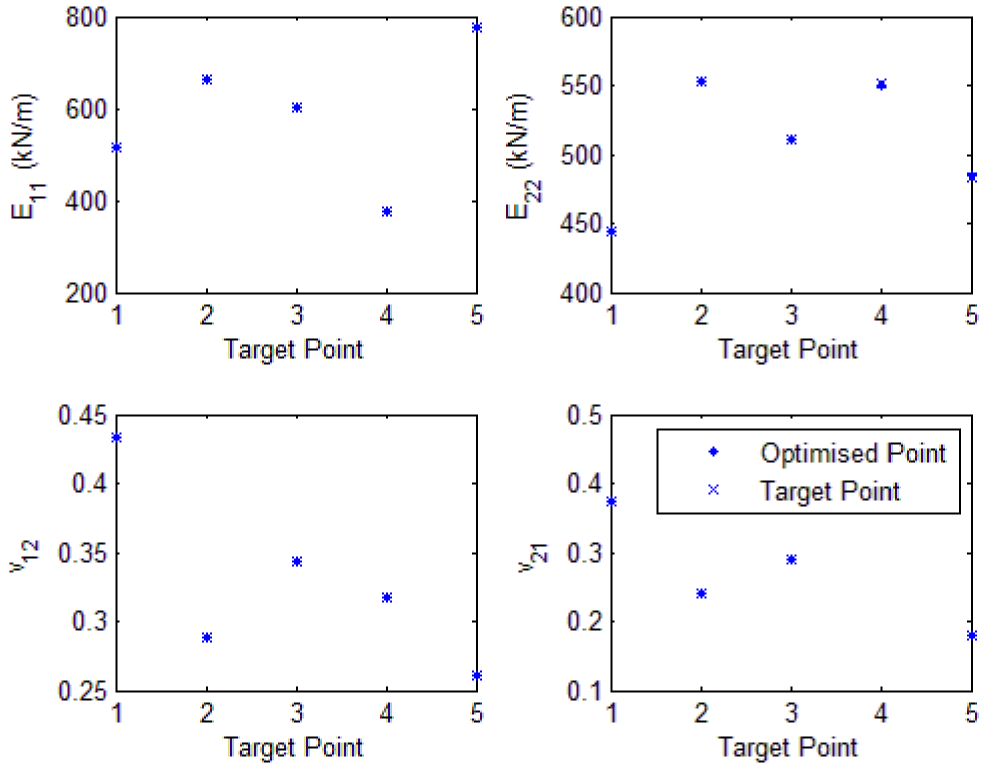

Figure 5: Results of the optimisation for the feasible solution

The optimisation for the feasible values of stiffness and poisons ratio produces good results (Figure 5). Target points 4 and 5 in the plot of E_{22} results show some slight deviation from the targets. In reality this small error, whilst observable in the figure, equates to a difference of 0.89kN/m and 0.90kN/m respectively. This is as a result of the slight deviation from the original geometry that was found. A higher accuracy requirement on the solver may produce more accurate results, but would be more computationally expensive, taking longer.

3.4 Comparison with measured fabric parameters

Target values of stiffness and poisons ratio were calculated from biaxial test data produced from a fabric with the geometry set out in Table 1. The targets are shown in Table 3, along with the numerical results of the optimisation. The points to be analysed were chosen from areas on the response surface that did not include flattening in one of the principle directions. This flattening leads to unexpectedly large or small results when analytical or numerical derivatives of the surface are calculated to give targets. Therefore similar targets to those used in the previous test could not be used in this instance.

Table 3: Measured targets found at the applied loads P1 and P2

		Point 1	Point 2	Point 3	Point 4	Point 5
Targets	E_{11} (kN/m)	700	799	794	668	596
	E_{22} (kN/m)	748	875	799	681	621
	v_{12}	0.218	0.170	0.197	0.114	0.138
	v_{21}	0.305	0.288	0.234	0.379	0.412
Results	E_{11} (kN/m)	552	652	604	611	591
	E_{22} (kN/m)	676	811	746	837	829
	v_{12}	0.248	0.153	0.196	0.145	0.152
	v_{21}	0.331	0.203	0.261	0.204	0.220
% differences	E_{11}	-21.1	-18.5	-24.0	-8.5	-0.8
	E_{22}	-9.6	-7.3	-0.5	22.9	33.4
	v_{12}	13.9	-10.1	-6.6	27.0	10.1
	v_{21}	8.6	-29.7	11.6	-46.3	-46.5
Applied Load	P_1 (kN/m)	10	20	14	12	10
	P_2 (kN/m)	10	20	14	16	14

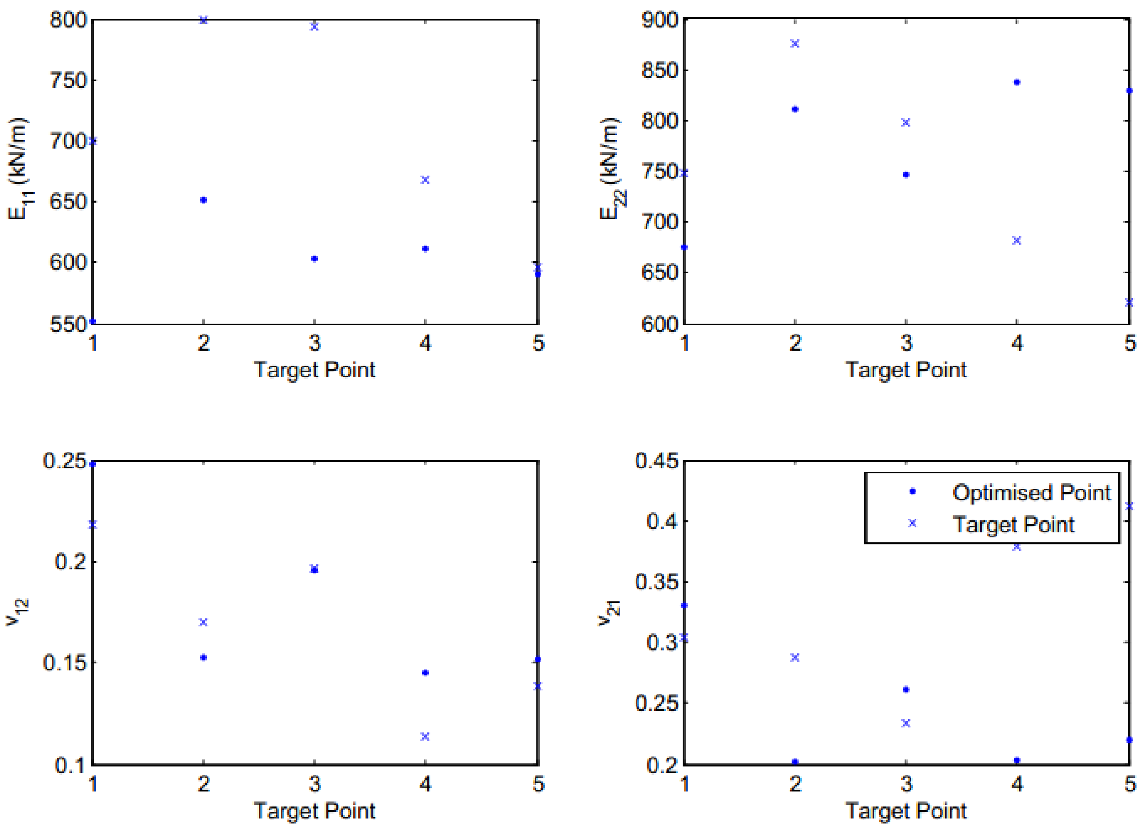

Figure 6: Results of the optimisation for the measured targets

No perfect solution could be found through the optimisation for the measured targets (Figure 6). Although no perfect solution could be found Figure 6 does show how close the solutions found were to the targets. Table 4 shows the geometric solution found against the geometry of the original fabric.

Table 4: Optimised geometry for measured targets

Variable	Geometry from which targets are calculated	Optimised geometry
A_1 (mm)	0.069	0.428
A_2 (mm)	0.207	1.861
Θ_1 (Rad)	0.106	0.316
Θ_2 (Rad)	0.189	0.130
L_1 (mm)	0.645	1.039
L_2 (mm)	1.082	0.210
r_1 (mm)	0.162	0.033
r_2 (mm)	0.114	0.334
w_1 (mm)	0.786	0.254
w_2 (mm)	0.673	1.021
E_1 (kN/m)	860	925
E_2 (kN/m)	710	946
E_k (kN/m)	30	19
v_k	0.3	0.3

The optimised geometry is clearly not the same as the geometry of the fabric from which the targets were derived. The original set of targets may be unobtainable for the sawtooth method with the constraints currently placed on the solution. The constraints (maximum and minimum values of geometric properties, and the constraints on the deformation stated in equation 2) currently being used are very broad to encompass extremes of realistic fabrics. These would be further constrained for more specific and realistic designs.

When the targets are allowed to vary slightly (5%) from the initial input targets a far more successful optimisation is performed.

6 DISCUSSION

The sawtooth model provides a reasonable prediction of fabric behaviour with the model's deviation from the mean of the strain range of a real fabric being between 5.3 and 5.9%[4] (Figure 7).

The method developed offers close correlation between results for feasible targets. This good fidelity was predicted, as the optimisation equations were developed using the sawtooth model, but demonstrates the utility of the method. Therefore the optimisation works by finding the solutions available from all possible response planes of the sawtooth model, and should eventually find a solution for targets that originally existed on this plane. This does, importantly, show that the method being employed to find the targets is working.

The error found in the optimised geometry for the targets measured from biaxial data can be explained by the difference in the response planes of the real fabric and the sawtooth's prediction of that fabric's response. Figure 7 shows the difference in the response planes of sawtooth and the real fabric when a sawtooth model is run using the geometry of the real fabric. These two sets of response planes, whilst similar, are clearly not the same. Over and under prediction of strain will also affect result.

It was unlikely at the outset that the solver would find a solution that perfectly matched the real fabric's geometry. It is also therefore possibly the case that no feasible solution exists for the sawtooth model where the targets stated in Table 3 could be achieved within the constraints placed on the model. Future work will be needed to demonstrate how much inaccuracy is inherent in the process, and therefore must be expected when attempting to design the geometry of 'real' fabrics.

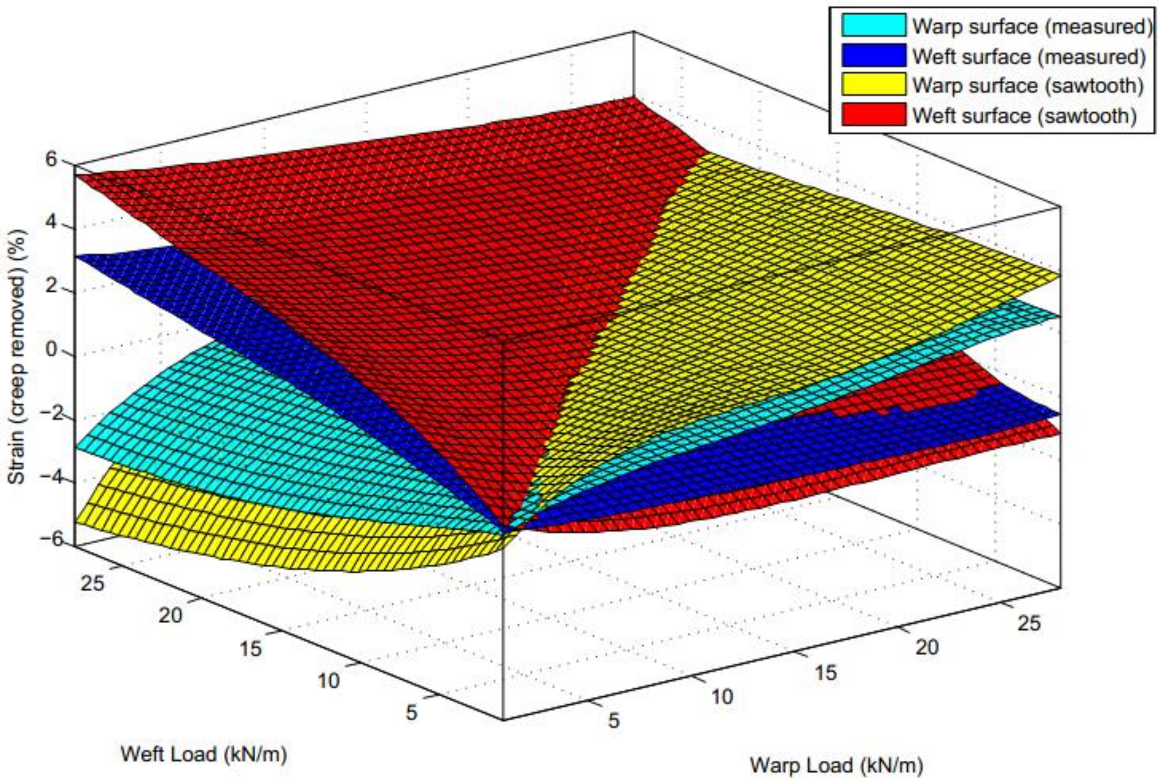

Figure 7: Response surfaces for the sawtooth model and measured response for one geometry

5 CONCLUSIONS

- The accuracy of the optimisation method with regards to known feasible targets derived from the sawtooth model is good.
- The methodology is slower than hoped as the calculation of $\frac{\Delta\varepsilon_{1,2}}{\Delta\varepsilon_{2,1}}$ must be completed after each iteration.

- The accuracy of the optimisation method with regards measured targets derived from real fabric data is acceptable at this stage of development. The actual accuracy of the optimised geometry for the new targets is unknown as it is not currently possible within the bounds of this work to produce a bespoke fabric to be tested.
- It is possible that for some targets multiple solutions exist and that for others no solutions exist. The latter has been shown through the results of the measured target optimisation, but the former is as of yet unproven.
- Allowing small amounts of variation from the target may drastically improve the model’s utility and allow for a Pareto front of possible solutions to be found.

6 FURTHER WORK

Further work is on-going to allow the optimum design of a fabric’s shear response characteristics as well as biaxial response to loads. The inherent uncertainty in the manufacturing process, and the discrete nature of some parameters, will also be considered and methods for the calculation of the effect of such variability incorporated into future models. In addition it is necessary to further check the inherent inaccuracy of the model when compared to real results obtained through tests. Other possible implications of the model must be further investigated. And the effect of varying one parameter on the optimised result will also be investigated.

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